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Dynamic Stability of a Compressible Cylindrical Fluid Under Axial Magnetic Influence

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ABSTRACT: The axisymmetric magnetohydrodynamic (MHD) stability of a compressible fluid cylinder subjected to electromagnetic and inertial forces is investigated. A general eigenvalue equation is derived and analyzed analytically, with numerical results used to validate the theoretical predictions. In the absence of internal and external electromagnetic forces, the system is influenced solely by capillary effects. The analysis shows that the model is unstable within the range $0 < x < 1$, while it becomes stable for $1 < x < \infty$. This suggests that instability occurs only for small axisymmetric disturbances, whereas the system remains stable in all other regions. At sufficiently high magnetic field strengths, the model exhibits complete stability across all wavelengths. Additionally, fluid compressibility is observed to have a stabilizing effect.

***Key words**: Hydrodynamic stability, rotating fluids, compressible fluids.*

INTRODUCTION

After studying a liquid column's stability, the authors came up with the dispersion relation. They looked at the kind of disturbance on the liquid jet's capillary instability boundary [1-4]. These pieces have been expanded [5–14]. Since the velocity is no longer solenoidal (i.e., ∇u 6=0) and the fluid's density is not uniform, the current work differs from earlier studies. This paper examines the stability of a complete fluid cylinder with a radius of R0 that is permeated by an axial magnetic field and has surface tension for all perturbation modes. It is assumed that the fluid is completely conducting, compressible, and inviscid. The magnetic field permeates the fluid.

FORMULATION OF THE PROBLEM

We investigate the stability of a complete fluid cylinder with a radius of R0 that is permeated by an axial magnetic field and has surface tension for all perturbation modes. It is assumed that the fluid is viscous, compressible, and perfectly conducting. The magnetic field penetrates the fluid, which has a density of ρ.

$$\underline{H}_0 = (0, 0, H_0).$$

(1)

The magnetic field is thought to permeate the surrounding tenuous medium around the cylinder.

$$\underline{H}_0^{vac} = (0, 0, \alpha H_0).$$

(2)

The capillary, gradient pressure, and electromagnetic forces are all being influenced by the fluid. Along the fluid-tense contact, the capillary force which results from the curvature pressure acts.The fluid's basic MHD equations under the current conditions are as follows.

$$\rho \frac{d\underline{u}}{dt} = -\nabla P + \mu(\nabla \times \underline{H}) \times \underline{H},$$

(3)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0, \qquad P = C\rho^{\gamma},$$

(4)

$$\frac{\partial \underline{H}}{\partial t} = \nabla \times (\underline{u} \times \underline{H}), \qquad \nabla \cdot \underline{H} = 0.$$

(5)

The fundamental equations in the vacuum zone are

$$\nabla \cdot \underline{H}^{vac} = 0,$$
$$\nabla \times \underline{H}^{vac} = 0.$$

$$P_s = S(\nabla \cdot \underline{n}_s).$$

(6)

Ps is the curvature pressure, C is a constant, S is the surface tension coefficient, ns is a unit outward vector normal to the cylinder interface, u and P are the fluid velocity vector and kinetic pressure, µ and H are the magnetic field permeability coefficient and magnetic field intensity, respectively, and Hvac is the magnetic field intensity in the vacuum region.

$$\underline{n}_s = \frac{\nabla F(r,\varphi,z,t)}{|\nabla F(r,\varphi,z,t)|},$$

The normal vector ns, the unit, is provided by

(7)

A condition of unperturbed The distribution of unperturbed pressure is ultimately provided by

$$P_0 = \frac{S}{R_0} + \frac{\mu H_0^2}{2}(\alpha^2 - 1), \qquad \frac{\mu H_0^2}{2}(\alpha^2 - 1),$$

(8)

where (S/R0) is the capillary force contribution, which results from the fluid cylinder's internal and external electromagnetic forces.

SOLUTIONS AND LINEARIZATION

For small departure from the unperturbed state, every physical quantity $Q(r,\varphi,z,t)$ could be expressed as .

$$Q(r,\varphi,z;t) = Q_0(r) + \varepsilon_0(t) Q_1(r,\varphi,z)$$

(9)

Where Q_1 stands for P ,$\underline{U}$, V^{ι} ,V^{e} , the amplitude of perturbation $\varepsilon(t)$ at time t is

$$\varepsilon(t) = \varepsilon_0 \exp(\sigma t)$$

(10)

Where (σ)is the growth rate of the instability or rather the oscillation frequency if ($\sigma = iw \text{ with } i = \sqrt{-1}$) is imaginary and ε_0 is the amplitude at $t = 0$. The perturbed radii distances f the gas cylinder is given by where $r = R_0 + \varepsilon_0 \, exp(\sigma t + i(kz + m\phi))$

(11)

Where (k) is the longitudinal wave number and (m an integer) is the transverse wave number. The second term on the right-hand side of equation (11) represent the surface wave elevation normalized with respect to R_0 and measured from the equilibrium position. The linearized perturbation equation deduced from the fundamental equations (1)- (4) given by

$$\Pi_1 = A I_0(\eta r) \exp[ikz + \sigma t],$$

$$\psi_1 = B K_0(\eta r) \exp[ikz + \sigma t],$$

$$\underline{H}_1^{vac} = B \nabla [K_0(\eta r) \exp[ikz + \sigma t]],$$

ANALYSIS OF PERTURBATIONS

For small departure from the unperturbed state, every physical quantity $Q(r,\varphi,z,t)$ could be expressed as .

$$Q(r,\varphi,z;t) = Q_0(r) + \varepsilon_0(t) Q_1(r,\varphi,z) \qquad (9)$$

Where Q_1 stands for P ,$\underline{U}$, V^{ι} ,V^{e} , the amplitude of perturbation $\varepsilon(t)$ at time t is

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$$\frac{\partial \underline{U}_1}{\partial t} + (\underline{U} . \underline{\nabla})\underline{U}_1 = -\underline{\nabla}\Gamma + 2(\underline{U}_1 \wedge \underline{\Omega}) \qquad (12)$$

$$\Gamma = \frac{P_1}{\rho} - V_1^1 \qquad (13)$$

$$\underline{\nabla} . \underline{U}_1 = 0 \qquad (14)$$

$$\nabla^2 V_1^i = 0 \qquad (15)$$

$$\nabla^2 V_1^e = 0 \qquad (16)$$

This system of equation is simplified on using the time dependence as given above by (10). From the view point of the linear theory and based on the linear perturbation technique, every perturbed quantity can be expressed as $exp(\sigma t + i(kz + m\phi))$ times an amplitude function of r . Consequently, on solving (12) we obtain

$$u_{1r} = \frac{-(\sigma+ikUe^{\omega t})}{\rho[(\sigma+ikUe^{\omega t})^2+4\Omega^2]} . \frac{\partial\Gamma}{\partial r} + \frac{4im\Omega^2}{\rho[(\sigma+ikUe^{\omega t})^2+4\Omega^2]} . \frac{\Gamma}{r} \qquad (17)$$

$$u_{1\phi} = \frac{2}{\rho[(\sigma+ikUe^{\omega t})^2+4\Omega^2]} . \frac{\partial\Gamma}{\partial \phi} + \frac{im\Gamma}{r(\sigma+ikUe^{\omega t})} .[\frac{4\Omega^2}{\rho[(\sigma+ikUe^{\omega t})^2+4\Omega^2]} - 1] \qquad (18)$$

$$u_{1z} = \frac{-ik\Gamma}{(\sigma+ikUe^{\omega t})} \qquad (19)$$

From equation (14) and (17,18,19) we get

$$\frac{d^2\Gamma}{dr^2} + \frac{1}{r}\frac{d\Gamma}{d r} + \left(q - \frac{m^2}{r^2}\right) = 0 \qquad (20)$$

$$q = k^2[1 + \frac{4\Omega^2}{(\sigma+ikUe^{\omega t})^2}] \qquad (21)$$

The solution of equation (20) is given by

$$\Gamma = A\, J_m(qr) exp(i(kz + m\phi + \sigma t)) \qquad (22)$$

Where A is an arbitrary constant to be determined and J_m the ordinary Bessel function of first kind of order m. Similarly equation (15) and (16), based on the linearzed theory, are solved and first order perturbation V_1^i and V_1^e are given by

$$V_1^i = B\, I_m(kr) exp(i(kz + m\phi + \sigma t)) \qquad (23)$$

$$V_1^e = C\, K_m(kr) exp(i(kz + m\phi + \sigma t)) \qquad (24)$$

Where I_m and K_m are modified Bessel function of order m and B, C are constantsof integrations to be determined.

BOUNDARY CONDITION

The Solution represented by equations (22) – (24) must satisfy certain boundary conditions. Under the present circumstances these conditions can be given as follows.

(i) Kinematics boundary condition states that " The normal component of the velocity U_{1r} vector must be compatible with the velocity of the particles of the boundary surface at $r = R_0$

$$U_{1r} = \frac{\partial r}{\partial t} = \frac{\partial \Gamma}{\partial r} \qquad (25)$$

(ii) The gravitational potential and its derivatives must be continuous across the surface.

(iii) The normal component of the total stress tensor must be continuous across the boundary surface from which we have the following dispersion relation:

$$\frac{(\sigma+ikUe^{\omega t})[(\sigma+ikUe^{\omega t})^2+4\Omega^2]}{[J_m(y)(\sigma+ikUe^{\omega t})+2imJ_m(y)]} J_m(y) = 2\pi G\rho - \Omega^2 - 4\pi G\rho K_m(x) I_m(x) \qquad (26)$$

Where x is, the dimensional wavenumber, given by $x = kR_0$ and y is defiend by $y = qR_0$.

Equation (26) is the dispersion relation of gravitational streaming oscillating rotating fluid cylinder surrounded by self-gravitating vacuum. It relates the growth rate σ with the streaming oscillating velocity $Ue^{\omega t}$, angular velocity Ω, the wave numbers x, y, m and other parameters ρ, G and R_0.

STABILITY DISCUSSION

It is preferable to examine the behaviors of the Bessel functions as well as the compound functions included in the relation before talking about the system's ordinary stability, marginal stability, and instability (26). Because of the recurrence relations (see Stegun and Abramowitz [12]),

$$2\, I_m^{`}(x) = I_{m-1}(x) + I_{m+1}(x), \qquad (27)$$

$$2\, K_m^{`}(x) = -K_{m-1}(x) - K_{m+1}(x) \qquad (28)$$

Because $I_m(x)$ is monotonic increasing and positive definite $I_m(x) > 0$ for all modes of perturbation $m \geq 0$ and nonzero values of $x \neq 0$, while $K_m(x)$ is monotonically decreasing but never negative, i.e., $K_m(x) > 0$ we may show that

$$\grave{I}_m(x) > 0, \qquad \grave{K}_m(x) < 0 \tag{29}$$

Also for $m \geq 1$ for all values of $x \neq 0, we\ have$

$$2I_m(x)K_m(x) < 1 \tag{30}$$

For non-rotating ($i.e\ \Omega = 0$) and non-streaming fluid ($i.e\ U = 0$) the dispersion relation (26) reduces to that of Chandraskhar[13] . Moreover if we put $m = 0$, $\Omega = 0$, $U = 0$ in (26) we recover the relation derived their relation by using the principle of Fermi .
In absence of the streaming ($i.e\ U = 0$), the dispersion relation (26) can be written in the dimensionless form

$$[y\mathrm{N}\acute{J}_m(y) + 2m\mathrm{M}J_m(y)]\left[K_m(x)I_m(x) - \frac{1}{2} + \mathrm{M}^2\right] + \mathrm{N}\left(\mathrm{N}^2 + 4\mathrm{M}^2\right)J_m(y) = 0 \tag{30}$$

Where the dimensionless quantity M, N we defined as follow

$$\mathrm{N} = \frac{\sigma}{\sqrt{4\pi G\rho}}, \qquad \mathrm{M} = \frac{\Omega}{\sqrt{4\pi G\rho}} \tag{31}$$

If $x = 0$ the dispersion relation (30) takes the simpler form

$$\mathrm{N}^2 - 2\mathrm{MN} + m\left(\mathrm{M}^2 - \frac{1}{2}\right) + \frac{1}{2} = 0\,. \tag{32}$$

Hence we get

$$\mathrm{N} = \mathrm{M} \pm \sqrt{(\mathrm{m} - 1)(\frac{1}{2} - \mathrm{M}^2)}. \tag{33}$$

A neutral mode of oscillation is obtained if

$$\mathrm{M} = \pm \frac{\sqrt{(m-1)}}{\sqrt{2m}}, \tag{34}$$

It is clear that the angular velocity must satisfy the following $\sqrt{2}\,\mathrm{M} \leq 1$ **(35)**

Neglecting the rotation effect the dispersion relation (26) reduces to

$$(\sigma + ikUe^{\omega t})^2 = \frac{4\pi G\rho x\dot{I}_m(x)}{I_m(x)}\left[I_m(x)K_m(x) - \frac{1}{2}\right]. \tag{36}$$

NUMERICAL DISCUSSIONS

in order to confirm the analytical results regarding the impact of various forces acting on the current model. The numerical discussion of the dispersion relation (26) for the ax symmetric perturbation mode is shown to be crucial. We must rephrase this dispersion relation in non-dimensional form in order to accomplish so, so we can enter it into the computer to perform the numerical calculation. Additionally, if we apply m=0 to equation (36) we get

$$(\sigma + ikUe^{\omega t})^2 = \frac{4\pi G\rho x I_1(x)}{I_0(x)}\left[I_0(x)K_0(x) - \frac{1}{2}\right]. \tag{37}$$

CONCLUSION

1. In both the ax symmetric and non-ax symmetric modes, streaming has the effect of reducing the stable states.

2. The self-gravitating force is stabilizing for all other perturbations, but only destabilizing in the symmetric mode (m=0) for a small range of wave numbers.

3. When the destabilizing propensity of the model is reduced and suppressed, its stability behavior is revealed.
4. This phenomenon is intriguing from an academic perspective and during geological drilling in the earth's crust since we have superposed gas-oil layer mixed fluids.

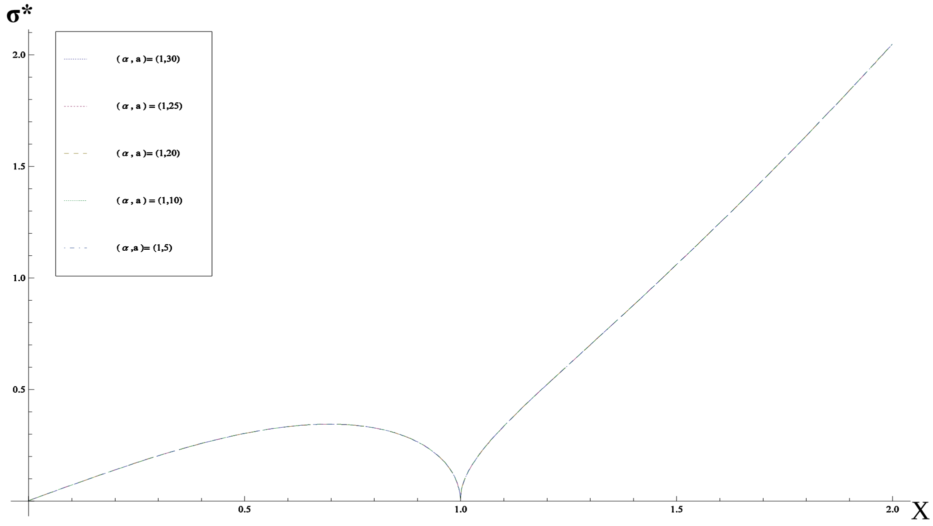

Figure (1): Magnetodynamic stable and unstable domains for a compressible fluid as H_0 /H_G=0

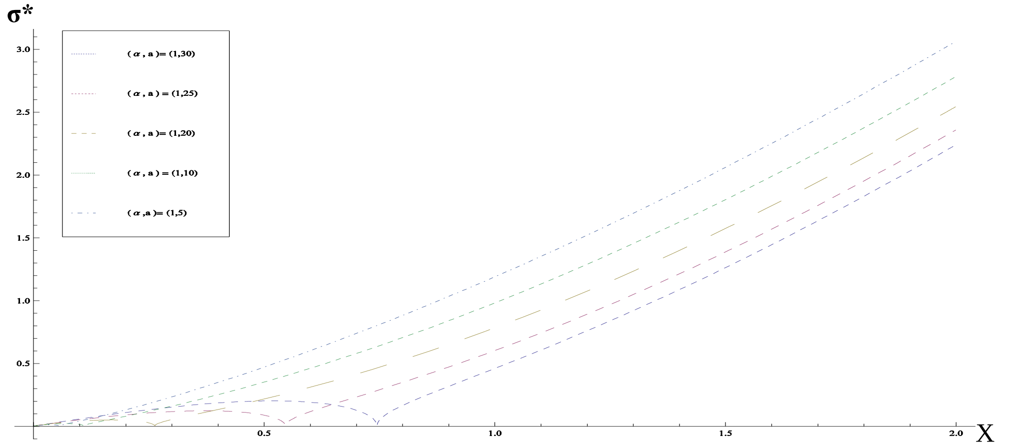

Figure (2): Magnetodynamic stable and unstable domains for a compressible fluid as H_0 /H_G=0.4

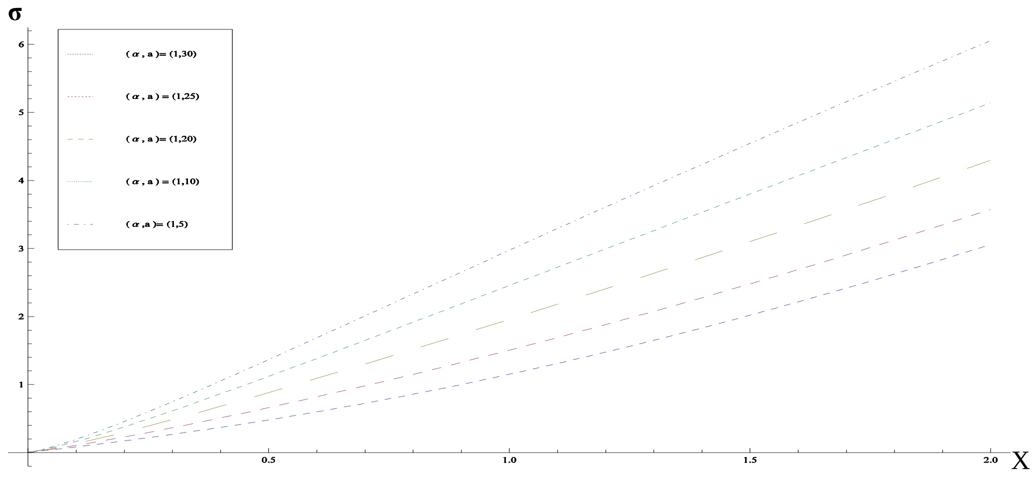

Figure (3): Magnetodynamic stable and unstable domains for a compressible fluid as H_0 /H_G=1.0

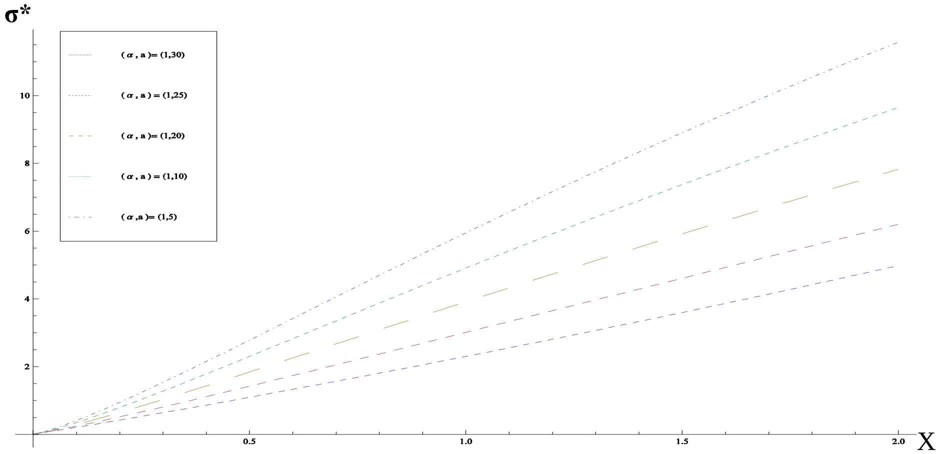

Figure (4): Magnetodynamic stable and unstable domains for compressible fluid as H_0/H_G=2.0

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